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Industrial spezialization, trade, and labour market dynamics in a multisectoral model of technological progress

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FLOWENLA Discussion Paper

Industrial spezialization, trade, and labour market dynamics in a multisectoral model of technological progress

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ABSTRACT

The issue of the impact of trade on economic performance and labour markets has been intensively discussed in the recent literature on trade liberalisation and globalisation where the debate was mainly about identifying the relative impact of trade and technology. The bulk of the existing literature in this area employs almost without exception a static Heckscher-Ohlin framework which seems not to be a suitable tool for analysing the ongoing dynamics. This paper presents a dynamic multi-sectoral framework with heterogenous labour to explore the issue of trade liberalisation and sectoral catching-up in productivity levels. The model is basically an input-output framework with Schumpeterian features; the latter are modelled as the impact of transitory rents which result from uneven productivity growth and technological catching-up upon the price and quantity systems of the trading economies. Relative productivity and wage rate dynamics across sectors determines the comparative costs and the dynamics of trade specialisation. In the appendix the equilibrium solutions of the model are derived.

Keywords: trade liberalisation, economic integration, labour markets, simulation, economic dynamics, growth
JEL-Classification: C62, C63, C67, D57, F15, F17

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INDUSTRIAL SPECIALIZATION, TRADE, AND LABOUR MARKET DYNAMICS IN A MULTISECTORAL MODEL OF TECHNOLOGICAL PROGRESS

Robert Stehrer [1]

1 Introduction

This paper presents a dynamic multisectoral model to study the effects of technological progress, catching-up and trade liberalisation on the labour market performance of different skill-types of workers in advanced and catching-up economies. The issue of the impact of trade on labour markets in the more advanced economies was widely discussed at the beginning of the 1990's, when a number of free trade agreements (especially the NAFTA between the US, Canada and Mexico) came into being. This debate focused mainly on the impact of developing countries and exporters of low-skill intensive goods on the relative wages of skilled to unskilled workers in the more advanced countries, especially the US. In the debate mainly the static Heckscher-Ohlin framework was used. On the one hand, especially Leamer (1994, 1996) and Wood (1995) argued that trade liberalisation was the main reason for the worsened labour market position of the unskilled workers. This explanation was, on the other hand, criticised e.g. by Lawrence and Slaughter (1993) and especially Berman et al. (1994). The latter pointed to skill-biased technological progress as the main explanation for the labour market positions of low skilled workers. Theoretical and empirical studies then focused on the relative impact of trade liberalisation versus

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technological progress. In most studies technological progress was found to have the most important impact on the labour market performance of the lower-skilled versus the higher skilled workers (measured either in relative wages or relative unemployment rates).

From the viewpoint of this paper there are several drawbacks in analysing the issue of trade and labour markets with the models mentioned above. The main criticism of the Heckscher-Ohlin framework concerns on its static nature and on the assumption of equal technologies across countries. Both assumptions seems to be at odds with economic 'realities'. In the model presented below, a Ricardian framework with catching-up in labour productivity levels is used but also differences in payments to factors of production can be introduced. The model allows the discussion of labour market effects on different skill-types of workers. In this sense the paper provides a dynamic multisectoral model of catching-up where the issues can be discussed in an integrated framework. The main focus of the simulation studies in this paper is to analyse the impact of 'shocks' (stemming from technology or trade liberalisation) and it thus deals mainly with non-steady-state and non-balanced growth and fluctuations. Although it is not the aim of the paper to study steady-state dynamics we show the properties of the balanced growth path in the appendix as this provides useful insights for discussion of the transitory dynamics.

The paper goes as follows: In the first part, the dynamic model used in the simulation studies is presented. This is first done for a closed economy with heterogenous labour which is then generalised for two trading economies. In the appendix the long-term dynamic equilibrium properties of the model are discussed. This is useful as the 'behaviour' of the model and some specific assumptions in the non-equilibrium transition phase become clearer if the long-term properties are accounted for. In section 3 a particular simulation study is presented. This simulation discusses a model with two trading economies, where one of them catches up in terms of productivity levels with the more advanced economy.

2 The model

2.1 Closed economy

In this section we present the detailed structure of the model, which is used afterwards in simulation studies. To be more explicit on the equations in this section we do not use matrix notation. The model is based on a paper by Landesmann and Stehrer (2000) and is an extension and modification of the model presented therein in a number of respects.

2.1.1 Technology

We start with a matrix of technical input coefficients, denoted by

$$\mathbf{A} = \begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}$$

which is assumed to be stable over time. Labour input coefficients for each skill-type of worker $z = 1, \dots, S$ in sector i is denoted by a^z_{li}. It is assumed that labour is used in fixed proportions. But these labour input coefficients a^z_{li} may decline over time at an exogenous rate $g_{a^z_{li}} \leq 0$ to a predetermined level $\bar{a}^z_{li}$:

$$\dot{a}^z_{li} = g_{a^z_{li}} \left(a^z_{li} - \bar{a}^z_{li} \right)$$

This formulation implies that $a^z_{li} > 0$ for all z and i thus, that each skill-type of labour is seen as a necessary input for the production of each good. Further the growth rate of labour productivity is going to 0, $\frac{\dot{a}^z_{li}}{a^z_{li}} \to 0$ for $t \to \infty$.[2]

2.1.2 Prices and rents

Prices Prices are modeled as adjustment to unit costs

$$\dot{p}_i = -\delta_{p_i} \left[p_i - (1 + \pi) c_i \right]$$

[2]The growth of labour productivity could also be endogenised. An endogenisation of technical progess in an input-output model which then exhibits endogenous growth dynamics was introduced for example in Los (2001). In this paper, however, we emphasize the *effects* of technological progress rather than the sources of productivity growth.

where $c_i = \sum_j p_j a_{ji} + b_i$ are the costs of production and $b_i = \sum_z w_i^z a_{li}^z$ denote the unit labour costs in a particular sector i. We assume that wage rates w_i^z need not be equal across sectors, although it is possible in general that wage rates for each particular skill-group equalise in the long run as we shall see below. The parameter $0 < \delta_{p_i} \leq 1$ gives the speed of adjustment of prices to (equilibrium) unit labour costs. There exists a long run mark up on prices π which is assumed to be equal across all sectors. This assumption leads to equal real per unit profits across sectors in the long run.

Rents As there is a constant mark-up on prices π there are long-run per unit profits r_i defined as

$$r_i = \pi c_i$$

As prices do not adjust immediately to unit costs plus mark-up there arise transitory rents depending on the speed of technological progress $g_{a_{li}^z}$ and the price-to-cost adjustment parameter δ_{p_i} and the dynamics of wages as we shall see below:

$$s_i = p_i - (1 + \pi)c_i = p_i - c_i - \pi c_i = p_i - c_i - r_i$$

2.1.3 Labour market

Wage rates Nominal wages are growing or falling for three reasons: First, transitory rents are partly distributed to workers; second, excess supply (demand) of workers in the labour market drives wages up or down; and third, we assume skill-specific wage equalisation across sectors. These three factors are formulated as follows:

$$\dot{w}_i^z = \kappa_{s_i^z} \frac{s_i}{\sum_z a_{li}^z} + \kappa_{u^z} u^z w_i^z + \kappa_{w^z} \frac{w_i^z - \bar{w}^z}{w_i^z} \qquad \text{with} \qquad \kappa_{s_i^z} = \kappa_{s_i} \frac{w_i^z}{\sum_z w_i^z}$$

$0 \leq \kappa_{s_i} \leq 1$ is the proportion of per unit (transitory) rents s_i paid to workers. The specification of the first term on the rhs of the wage equation implies that wage rates of different types of workers are absorbing a certain proportion of sector-specific rents (the latter are defined per unit of output). This means that wage rates can (temporarily) be different across sectors and skill-groups as rents are, in the first instance, distributed only to workers in the respective sector where the rents arise.

The second term on the rhs of the wage dynamics equation reflects the impact of unemployment on the dynamics of the wage rates ($\kappa_u^z \leq 0$). The unemployment rate is defined as $u^z = \frac{L^{S^z} - \sum_i a_{li}^z q_i}{L^{S^z}} = \frac{L^{S^z} - L^{D^z}}{L^{S^z}}$ where L^{S^z} and L^{D^z} denote labour supply and demand, respectively. Third, there is an impact on the wage dynamics if wage rates (for the same skill-type of worker) differ across sectors. This reflects the common assumption that wage rates get equalised across sectors because of labour mobility. The (weighted) average wage rate is defined as $\bar{w}^z = \frac{\sum_i L_i^{D^z} w_i^z}{\sum_i L_i^{D^z}}$. If the average wage $\bar{w}^z$ is higher than the sectorial wage w_i^z the wage in sector i will rise ($\dot{w}_i^z > 0$ for $\kappa_{w^z} < 0$), in the other case fall. This term works across all sectors. Thus in the formulation used in the simulations, there is a sector specific term and two economy wide terms having an influence on wage rates in each sector. Skill-specific wage differentiation can occur across sectors in the short run but wages are equalised, however, in the long run.

Labour supply Skill-specific labour supply L^{S^z} is assumed to adjust to labour demand according to

$$\dot{L}^{S^z} = \delta_{L^{S^z}} \left(L^{D^z} - L^{S^z} \right)$$

where $L^{D^z} = \sum_i a_{li}^z q_i$ and

$$\delta_{L^{S^z}} = \begin{cases} \delta_{L^{S^z}_{IN}} > 0 & \text{for } L^{S^z} > L^{D^z} \\ \delta_{L^{S^z}_{OUT}} \geq 0 & \text{for } L^{S^z} \leq L^{D^z} \end{cases}$$

This formulation implies that labour supply adjusts to labour demand if there is excess demand or excess supply of labour; adjustment occours at different rates, however. In the first case workers are entering the labour market, in the second case workers leave the labour market in case of unemployment, meaning that high unemployment leads to a falling participation rate.[3]

[3]This formulation guarantees that in the long-run growth equilibrium labour supply is growing at the same rate as labour demand. A referee pointed out that labour supply should also be linked to the growth (or the level) of (real) wage rates. I have not yet introduced this linkage into the model presented in this paper as then there may occur cycles in the adjustment process similarly to the Goodwin model (see Goodwin, 1986). For further reasons of this assumption with respect to the simulations see below.

2.1.4 Quantities

After the discussion of the price system the quantity system must be specified. Demand for goods consists of three different components which can be summarized in the following general demand equation:

$$\begin{aligned} q_i^D &= \sum_j a_{ij} q_j + \sum_j a_{ij} q_j \frac{\sum_k v_k q_k}{\sum_{kj} p_k a_{kj} q_j} + \alpha_i \sum_{j,z} \frac{w_j^z}{p_i} a_{lj}^z q_j \\ &= \sum_j a_{ij} q_j + q_i^I + q_i^F \end{aligned}$$

with $\sum_i \alpha_i = 1$. The first term is demand for intermediate goods used in production, the second term is (net) investment demand (financed - by assumption - out of profit income) and the third term reflects demand out of workers income (used for consumption). g denotes the growth rate of the economy. $q_i^I + q_i^F$ is investment and consumption demand for good i. We discuss each of the two items separately.

Consumption demand First we assume that that workers do not save their income (or spend all money on consumption goods). For consumer demand we assume that the nominal shares of consumption α_i are constant and $\sum_i \alpha_i = 1$. Or, stated differently, consumers maximize a Cobb-Douglas utility function, $U = \prod_i q_i^{\alpha_i}$, from which this kind of consumer behaviour results. Of course, any demand system - e.g. Stone-Geary, CES utility functions, the AIDS demand system, or Dixit-Stiglitz type - which determines nominal shares for given income and prices could be used instead of the simple Cobb-Douglas system. The importance of non-linear Engel curves are discussed in Pasinetti (1981) where commoditiy-specific Engel curves are introduced. Another general form of demand equations allowing for non-linear Engel curves is introduced in Verspagen (1993), chapter 7. The specification above assumes that the nominal consumption shares are equal for all skill groups. In a more general setting the nominal shares may depend on skill- and sector-specific wage income thereby using demand systems with non-linear Engel curves.

Investment demand We assume that profits and rents which are not distributed to workers $v_i = \big((1-\kappa_{s_i})s_i + r_i\big)q_i$, are entirely used for investment and that investment cannot be negative. Total rents plus profits in nominal terms available for investment in the economy are then given by $\sum_i v_i q_i$. In the following it suffices to model the demand for goods for investment purposes in sector i in the aggregate. In the appendix we show that demand for investment purposes for good i is given by

$$q_i^I = \sum_j a_{ij} q_j \frac{\sum_k v_k q_k}{\sum_{kj} p_k a_{kj} q_j}$$

This specification describes the aggregate outcome of investment decisions and do not model the specific investment decisions at the industrial level. Further it is assumed with this formulation that investment structure adjusts immediately to output structure which is itself determined by the coefficient matrix and the structure of consumption. As the structure of the economy changes over time (e.g. due to changes in relative prices when assuming the simple Cobb-Douglas demand structure) the structure of investments also has to change. The assumption above implies that the structure of capacities (and hence net investments) adjusts continously to the structure of demand to enable the economy to expand on an equilibrium growth path. The expansions of capacities (and hence net investment rates) are then given by $g_i = \frac{q_i^I}{\sum_j a_{ij} q_j} = \frac{\sum_k v_k q_k}{\sum_{kj} p_k a_{kj} q_j}$ which is equal for all sectors i.

Supply of goods The supply of goods is modeled as an adjustment process where supply adjusts to demand in a growing economy with:

$$\dot{q}_i = (1+g) \sum_j \tilde{a}_{ij} \left(q_j^I + q_j^F \right) - q_i$$

where $\tilde{a}_{ij}$ denotes a typical element of the Leontief inverse $[\mathbf{I} - \mathbf{A}]^{-1}$. The rationale for this specification is as follows: At each point in time there exists a demand vector, $q_i^I + q_i^F$. For the system to be able to produce these quantities the (direct and indirect) intermediate demand for the production of each good must be taken into account, which is done by the Leontief inverse. Further each sector is able to grow only if there is a positive investment in this sector, which amounts - in this model with circulating capital only - to a growing stock of intermediate inputs. In the appendix A we show that in equilibrium (i.e. steady state balanced growth) the investment structure as defined above guarantees that the economy grows with $g^* = \frac{\pi}{1+\pi} \frac{\sum_i p_i q_i}{\sum_{ij} p_i a_{ij} q_j}$.[4]

2.2 Integrated economies

The next step is to introduce more countries and especially international relationships between these countries. First of all, all the variables have to be indexed for the different countries $1, \ldots, C$. In this paper we assume that the exchange rate between the trading economies c and s is set to 1 and there are no changes over time. Further the various economic relationships between the countries have to be specified where we proceed in four steps.

2.2.1 Exports and imports for consumption

For consumer demand we adopt a specification which is similar to the specification in the closed economy case. Demand for consumption goods in country c now depends also on income in the other countries $s = 1, \ldots C$. For simplicity, we assume that a constant nominal share of wage income μ_i^{cs} in country c is spent on goods from countries $s = 1, \ldots, C$ with $\sum_s \mu_i^{cs} = 1$. Equivalently, a constant nominal share μ_i^{sc} of countries $s = 1, \ldots, C$ is spent on goods from country c. The nominal share of wage income in economy c spent on goods in economy s can then be written as $\alpha_i^{cs} = \alpha_i^c \mu_i^{cs}$ where $\sum_{s,i} \alpha_i^{cs} = 1$ is satisfied by assumption. This specific assumption means that the domestic and foreign good are not (or not seen as) perfect substitutes. In fact, the formulation used here implies a Cobb-Douglas utility function of the form[5] $U^c = \prod_{i,s} (q_i^s)^{\alpha_i^{cs}}$ where

[4]Given the specification, it occurs in the transitory phases that either demand exceeds supply or supply is larger than demand. Such disequilibrium quantities, however, turned out to be very small in the simulations below. We thus do not take into account any effect of excess demand or supply e.g. on prices, investment, consumption demand, nor do we specify rationing of consumption or investment patterns.

[5]Of course here again more flexible functions could be used which e.g. allow for home-bias effects, other price elasticities between foreign and domestic goods, etc. Further one could also introduce a specification that these nominal shares evolve gradually in the case of a sudden trade liberalisation.

the price elasticity equals −1. Demand for a particular good i in country c is then given by

$$\left(q_i^F\right)^c = \sum_s \sum_j \sum_z \alpha_i^{sc} \frac{w_j^{z,s}}{p_i^c} a_{lj}^{z,s} q_j^s$$

2.2.2 Investment flows

Investors have to make two decisions: First, in which country and sector to invest, and second in which country to buy the goods for investment. The decisions for these two decisions are guided by different considerations: The first one is motivated by relative per unit rents (and profits), the second by relative prices for purchases of investment goods. For simplicity, we assume that a constant share of nominal rents and profits in sector i of country c, $\left[(1-\kappa_{s_i}^x)s_i^c + r_i^c\right] q_i^c$, is invested in country s. The nominal share is denoted by ν_i^{cs} with $\sum_s \nu_i^{cs} = 1$. We do not specify the sector in which investment is taking place in country s but assume that foreign investment is structured as in the closed economy case in a way that the growth rate in country s is maximised. Equivalently, the nominal share of investment of country s in country c is given by ν_i^{sc}. Second, we have to specify the country where the goods for investment of country c in country s are purchased. Here we use a constant nominal shares assumption: ξ_i^{cr} with $\sum_r \xi_i^{cr} = 1$, denotes the nominal share of rents and profits accruing in sector i of country c spent for purchase of investment goods in country r. This implies a positive demand effect in country r.[6] The product $\nu_i^{cs}\xi_i^{cr}$ with $\sum_{s,r} \nu_i^{cs}\xi_i^{cr} = 1$ then denotes the nominal share of demand for investment goods country c spends in country r for investment in country s. By analogous reasoning as in the closed economy case demand for investment goods in country c is then given by

$$\left(q_i^I\right)^c = \sum_{r=1}^{C} \frac{\sum_j a_{ij}^r q_j^r}{\sum_{k,j} p_k^r a_{kj}^r q_j^r} \sum_{s=1}^{C} \sum_{k=1}^{N} \nu_k^{sr} \xi_k^{sc} t_k^s q_k^s$$

Similarly, one can calculate the inflow of physical capital into economy c in sector i with

$$\left(q_i^I\right)^c = \frac{\sum_j a_{ij}^c q_j^c}{\sum_{k,j} p_k a_{kj} q_j} \left[\sum_{k=1}^{N} \sum_{r=1}^{C} \nu_k^{rc} v_k^r q_k^r\right]$$

[6] As before, here again short-term imbalances are arising in the supply and demand for goods which, however, are quite small in the simulations.

Rearranging gives the equilibrium growth rate in country c by

$$g^c = \frac{\left(q_i^I\right)^c}{\sum_j a_{ij}^c q_j^c} = \frac{\sum_{k=1}^N \sum_{r=1}^C \nu_k^{rc} \upsilon_k^r q_k^r}{\sum_{k,j} p_k^c a_{kj}^c q_j^c}$$

Please note, that this formula collapses to the one in the closed economy case by setting $\nu_i^{rc} = 0$ for $r \neq c$ and $\nu_i^{rc} = 1$ for $r = c$.

2.2.3 Quantity dynamics

Given these assumptions on the consumption and investment behaviour in the international setting the demand for products in country r can then be written as:

$$\left(q_i^D\right)^c = \sum_j a_{ij}^c q_j^c + \left(q_i^I\right)^c + \left(q_i^F\right)^c \tag{2.1}$$

The supply adjusts to demand differential equation then becomes

$$\dot{q}_i^c = (1 + g^c) \sum_j \tilde{a}_{ij}^c \left(\left(q_j^I\right)^c + \left(q_j^F\right)^c\right) - q_i^c \tag{2.2}$$

2.2.4 Learning processes

Another international linkage of economic integration is that countries can learn from each other, meaning that technologically backward countries are catching-up with more advanced countries. The simplest modelling strategy, which is used in this paper, is that countries are catching-up to the leading country (or the productivity frontier). Different paths of catching-up processes were investigated in Landesmann and Stehrer (2001) and should not be repeated here. In the simulations above we assume that industries lying farther behind have relatively higher productivity growth rates (Gershenkron's 'advantage of backwardness' which is applied here at the industrial level; see also Landesmann and Stehrer (2001) for a theoretical discussion and empirical analysis). The specific equations for the catching-up processes are similar to the closed economy case:

$$\dot{a}_{li}^c = g_{a_{li}}^c \left(a_{li}^c - \bar{a}_{li}^L\right) \tag{2.3}$$

where $\bar{a}_{li}^L$ denotes the labour input coefficient of the technological frontier of the productivity leader. In a more sophisticated setting, the speed of catching-up could also depend on the country-wide or industry-specific skill-structure (relative to other countries), exogenously given learning parameters, the structure and volume of imports and exports and especially the flows of international investments.

2.2.5 International effects on prices

The last effect of international trade is that goods prices may equalise in the long run ('law of one price'). In the set-up of the model so far a long-term equilibrium could exist with persistent differences in prices, as the production structure may change to the equilibrium structure in each country and there is no effect on prices via excess supply or demand. In the following we therefore assume an exogenous trend for price equalisation. This alters the system of differential equations for prices which becomes now

$$\dot{p}_i^c = \delta_{p_i}^c \left[p_i^c - (1+\pi)c_i^c\right] + \delta_{\bar{p}_i}^c \frac{p_i^c - \bar{p}_i}{p_i^c} \tag{2.4}$$

where $\bar{p}_i = \frac{\sum_r q_i^r p_i^r}{\sum_r q_i^r}$ is a weighted average of the prices in the world market.

2.2.6 Trade balance and capital account

Imports and exports of consumption goods Using the notation introduced above exports of country c to country s are given by

$$\left(x_i^F\right)^{cs} = \sum_j \sum_z \alpha_i^{sc} \frac{w_j^{z,s}}{p_i^c} a_{lj}^{z,s} q_j^s$$

and imports of country c from country s are

$$\left(m_i^F\right)^{cs} = \sum_j \sum_z \alpha_i^{cs} \frac{w_j^{z,c}}{p_i^s} a_{lj}^{z,c} q_j^c$$

The nominal values can easily be calculated by multiplying with prices p_i^c and p_i^s, respectively, and total exports and imports for consumption can be calculated by summing over all trading partners.

Imports and exports of investment goods We assume that if country r is investing in country c and buying goods for this investment in country s this means imports of goods of country c from country s and, vice versa, if countries r are investing in country s and are buying goods for this investment in country c means exports of goods of country c to country s. Using the notation from above this means formally that

$$\left(m_i^I\right)^{cs} = \frac{\sum_j a_{ij}^c q_j^c}{\sum_{k,j} p_k^c a_{kj}^c q_j^c} \sum_r \sum_k \nu_k^{rc} \xi_k^{rs} v_k^r q_k^r \qquad for \qquad s \neq c$$

are imports of investment goods i of country c from country s (financed by capital flows from countries $r = 1, \ldots C$ to country c). Similarly the exports of investment goods i of country c to country s (financed by capital flows from countries $r = 1, \ldots C$ to country s) are denoted by

$$\left(x_i^I\right)^{cs} = \frac{\sum_j a_{ij}^s q_j^s}{\sum_{k,j} p_k^s a_{kj}^s q_j^s} \sum_r \sum_k \nu_k^{rs} \xi_k^{rc} v_k^r q_k^r \qquad for \qquad s \neq c$$

For calculating value terms one has to multiply imports by p_i^r and exports by p_i^c, respectively. Total imports and exports of country c can then be calculated by summing up over all trading partners.

Trade balance The trade balance can the easily be calculated by summing up exports and imports. Net exports are given by

$$b^c = \sum_{s(s \neq c)} \sum_i \left\{ p_i^c \left[\left(x_i^F\right)^{cs} + \left(x_i^I\right)^{cs} \right] - p_i^s \left[\left(m_i^F\right)^{cs} + \left(m_i^I\right)^{cs} \right] \right\}$$

Capital account Financial flows from country s to country c are given by $\sum_i \nu_i^{sc} v_i^s q_i^s$ and equivalently financial flows from country c to country s are $\sum_i \nu_i^{cs} v_i^c q_i^c$. Similarly to above total inflows and outflows of profits and rents can then by calculated by summing over s with $s \neq c$. The difference of these two items gives the capital balance.

Balance of payment surplus/deficit Finally we can calculate the balance of payment surplus or deficit as sum of the trade balance and the capital balance defined above:

$$BoP^c = b^c + \sum_{s(s \neq c)} \sum_k \left(\nu_k^{sc} v_k^s q_k^s - \nu_k^{cs} v_k^c q_k^c \right)$$

which is in a reduced form as we have not introduced monetary flows into the model. But one can show that $\sum_c BoP^c = 0$ which must hold by definition is satisfied.

3 Effects of international integration and catching-up

In this section a simulation study is presented which reflects the discussion on the effects of trade and technological progess on labour markets as discussed above.[7] We study the dynamics of two interacting economies. This is done in the following way: The more advanced country A is characterised by the set of parameters and starting values as shown in tables 3.1 and 3.2, respectively.[8] The starting values equal the (closed economy) equilibrium values. Further the catching-up economy B has also the same parameter values (with exception to one discussed below) but different starting values which will be discussed below. The parameters of international linkages have also to be specified.

3.1 The assumptions

Tables 3.1 and 3.2 summarise the assumptions made for both countries for parameters and starting values. Specifically we assume that both countries are equal in every respect with the exception of the influence of the average world price $\bar{p}_i$ on national prices. Here we assume that the less advanced country adjusts to world prices (or average prices $\bar{p}_i$) quite fast with $\delta^B_{\bar{p}_i} = 0.15$ whereas the prices of the more advanced countries are less influenced ($\delta^A_{\bar{p}_i} = 0.01$).[9] The parameter values for the international linkages are specified with μ_i^{sr} for the shares of consumption expenditures spent abroad which are assumed to

[7]The model was written and simulated in DMC (Medio, 1992). A Runga-Kutta algorithm is used for numerical simulations.

[8]A referee pointed out that it is common in the input-output literature to set values that the column sums of the intermediate input requirement matrix **A** and the compensations for making primary inputs add up to one which then implies $p_i = 1$ for all i. In this paper we do not set the values this way for three reasons: First, differences in prices play an important role in determining the demand for goods, and second, differences in price levels and relative prices are important in the dynamics of trade patterns in the integrated economy. Third, prices change over time because of changes in labour input coefficients (which changes the skill-compostion) and changes in skill-specific wages.

[9]This assumption can be justified in the following way: The starting values are set in a way that both countries are of similar size. The larger value of $\delta^r_{\bar{p}_i}$ for the less advanced country thus could also be seen as a parametrisation for differences in the size of countries. The less advanced country is more strongly influenced by the leader country (or the world market) than vice versa.

Parameter	Country A				Country B			
	Sector specific				Sector specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
a_{ii}	0.400		0.400		0.400		0.400	
a_{ij}	0.100		0.100		0.100		0.100	
π_i	0.000		0.000		0.000		0.000	
δ_{p_i}	0.100		0.100		0.250		0.250	
$\delta_{\bar{p}_i}$	0.010		0.010		0.150		0.150	
κ_{s_i}	0.100		0.100		0.100		0.100	
$\delta_{\beta_i^*}$	1.000		1.000		1.000		1.000	
α_i	0.500		0.500		0.500		0.500	
μ_i^{sr}	0.250		0.250		0.250		0.250	
ν_i^{sr}	0.250		0.250		0.250		0.250	
ξ_i^{sr}	0.250		0.250		0.250		0.250	
	Sector and skill specific				Sector and skill specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled
$\bar{a}_{li}^z$	1.000	2.000	0.500	2.000	1.000	2.000	0.500	2.000
$g_{a_{li}^z}$	0.000	-0.015	0.000	-0.015	-0.015	-0.015	-0.015	-0.015
	Economy wide, skill specific				Economy wide, skill specific			
	Skilled		Unskilled		Skilled		Unskilled	
$\delta_{L_{S^z}}(L^{S^z} \leq L^{D^z})$	1.000		1.000		1.000		1.000	
$\delta_{L_{S^z}}(L^{S^z} < L^{D^z})$	0.000		0.000		0.000		0.000	
κ_{u^z}	0.075		0.075		0.075		0.075	
κ_{w^z}	0.010		0.010		0.010		0.010	

Table 3.1: Parameter values used in simulations

be equal across sectors and countries. The same assumption was made for investment expenditures ν_i^{sr}. There are some differences across countries and sectors with respect to the starting values. Here especially the assumptions on labour productivity and wages for the two skill-types of workers are relevant, as all other starting values are influenced by them. Sector 1 is the more skill-intensive sector in both countries. Country B has lower productivity levels in both sectors. Specifically we assume that the skilled workers have equal labour productivity levels, whereas the labour input coefficient for unskilled workers in country B in sector 1 are three times higher and in sector 2 are two times higher than in country A. From this structure of labour input coefficients follows that country B has a comparative advantage (in terms of productivity) in sector 2, the low-skill labour intensive sector. Further we assume that the relative wage rates of skilled workers are lower in country A than in country B; the skill-specific wage rates are equalised across sectors in both countries. This again leads to the situation that country B has a comparative advantage in sector 2.

In the simulations below it was assumed, that there can be no excess demand for labour as labour is supplied with infinite elasticity and adjusts immediately to labour demand.[10] This assumption can be justified for two reasons: First, there is some evidence that shortage of labour has not acted as a constraint in the long run growth of economies (e.g. McCombie and Thirlwall, 1994). Further with application to actual catching-up processes of some countries a shortage of labour was never discussed as limiting factor, as either the labour supply responds sufficiently fast to the growth process or labour is available from other sectors (as for example in the model by Lewis, 1954). Second, from a modelling point of view, a constraint of labour supply would imply a further specific assumption on the distribution of labour across the sectors, which is especially a difficult problem when assuming more than one skill-type of workers.[11] One has to notice that

[10]Thus the production is not constrained by shortages in the supply of labour, although in the simulations we allow for a pressure on wage rates due to excess demand of labour via the unemployment term. This excess demand of labour results from the numerical solution of the system of differential equations. As labour supply adjusts rapidly to demand ($\delta_{L^{S^z}} = 1$ for $L^{S^z} \leq L^{D^z}$) this effect may not be very large.

[11]The limitations of (sectoral) growth due to a shortage of factors would of course be an interesting

Variable	Country A				Country B			
	Sector and skill specific				Sector and skill specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled	Skilled	Unskilled
a^s_{li}	1.000	2.500	0.500	2.500	1.000	7.500	0.500	5.000
w^s_i	1.000	0.200	1.000	0.200	0.600	0.100	0.600	0.100
$L^{D^s}_i$	1.000	2.500	0.614	3.068	1.000	7.500	0.650	6.501
	Sector specific				Sector specific			
	Sector 1		Sector 2		Sector 1		Sector 2	
ω_i	1.500		1.000		1.350		0.800	
p_i	2.857		2.143		2.543		1.757	
c_i	2.857		2.143		2.543		1.757	
r_i	0.000		0.000		0.000		0.000	
s_i	0.000		0.000		0.000		0.000	
q^I_i	0.000		0.000		0.000		0.000	
q^C_i	0.477		0.636		0.470		0.680	
q_i	1.000		1.300		1.000		1.292	
	Economy wide, skill specific				Economy wide, skill specific			
	Skilled		Unskilled		Skilled		Unskilled	
L^{D^s}	1.614		5.568		1.650		14.001	
L^{S^s}	1.614		5.568		1.650		14.001	
u^s	0.000		0.000		0.000		0.000	
	Economy wide				Economy wide			
g	0.000				0.000			
b	0.000				0.000			

Table 3.2: Starting values used in simulations

the assumption is more restrictive for skilled workers (which have to be educated) than for low- or unskilled workers.

Given these assumptions the prices p_i^c can be derived. The relative price of good 1 (the skill intensive good) is lower in country A than in country B. Given the structure of consumption and investment the relative output of good 1 is thus relatively higher in country A than in country B. The absolute price level is lower in country B for both goods.[12] Although this does not have an effect on the specialisation structures it will lead to a shift of aggregate demand to country B via the formulation of the expenditures abroad. Finally, this leads to the labour market outcome that in country A relatively more skilled workers are employed. As one can see, this structure of starting values captures Ricardian and factor endowments (or payments) characteristics, which are quite common in the literature. With respect to the evolution of labour productivity we assume that the less advanced country B, starts immediately with catching up to the labour productivity levels of the more advanced country, country A. The specific assumption of the dynamics of the labour input coefficients implies, that convergence is relatively faster in sector 1 than in sector 2 as there is more 'scope for learning' in sector 1 where the initial productivity gap is higher than in sector 2.[13] Further there is exogenous technical progress in country A, which is biased against the low-skilled workers. The resulting implications for the other variables as prices, output structure, wage rates, etc. are discussed in the following section.

3.2 Simulation results

In the following we shall now present the following stylised scenario: Both economies, which are starting from long-term autarkic equilibria, are 'shocked' by a sudden trade

topic in itself, but this is not a topic in this paper.

[12]This is mainly empirically motivated. The lower price level means that either wages are not exactly reflecting the productivity gap of country B for whatever reasons or reflect the undervaluation of the currency (which is not modeled explicitely).

[13]For empirical evidence of this pattern of a relatively faster catching-up in the higher-tech sectors see Landesmann and Stehrer (2001).

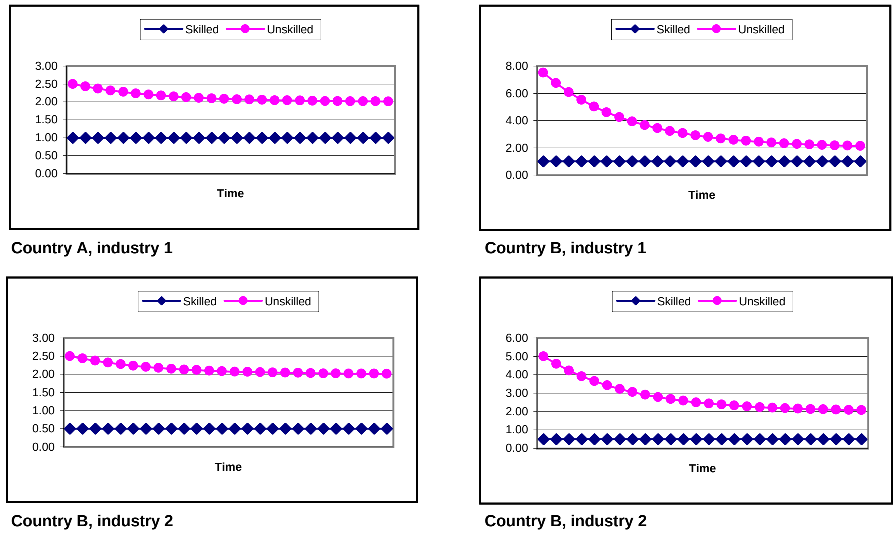


Figure 3.1: Labour input coefficients

liberalisation. At the same time technological progress occurs as described above.

The evolution of the labour input coefficients in both countries is drawn in figure 3.1.[14] The skill-biased technical progress in country A is equal in both sectors and has been discussed. The impact on unit labour costs is thus larger in sector 2, as this is the low-skill intensive sector. In country B technical progress (or convergence) is also biased against the low-skilled workers and is faster in sector 1, the skill-intensive sector, as the initial gap is larger than in sector 2. In this sense we have factor-biased technical change and simultaneously sector-biased technical change in country B.

From this evolution of labour productivity and relative wage dynamics, which is discussed below, results the dynamics of prices presented in figure 3.2. The absolute price level in both sectors in country A is falling due to the effect of changes in labour input coefficients and the incomplete nominal wage rate adjustment. Further there is a small impact of the lower prices of country B on prices in country A. The relative price of the skill-intensive good p_1/p_2 in country A is increasing. The reasons for this is that technical change is biased against the unskilled workers and thus the effect on prices in the unskill-intensive sector 2 is larger. Price levels in country B are first rising due to the adjustment

[14]The model is not calibrated in a way that 'time' should be understood in terms of years, etc.

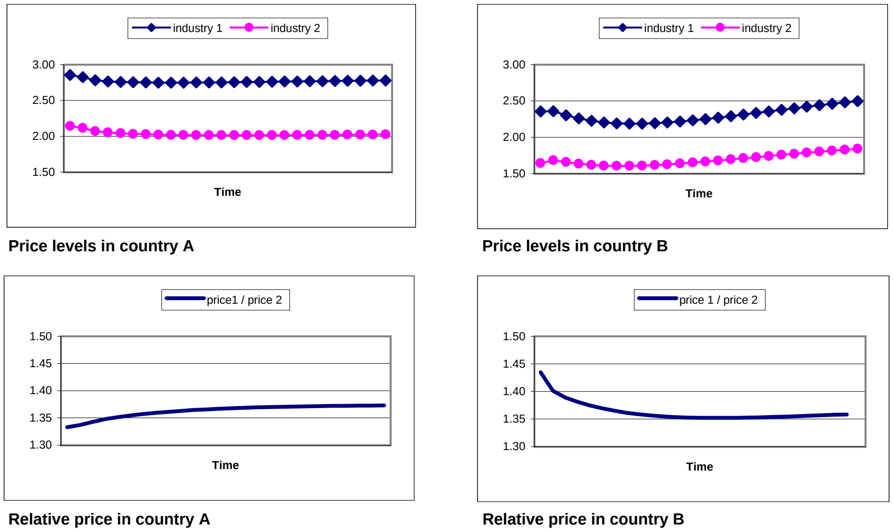


Figure 3.2: Prices

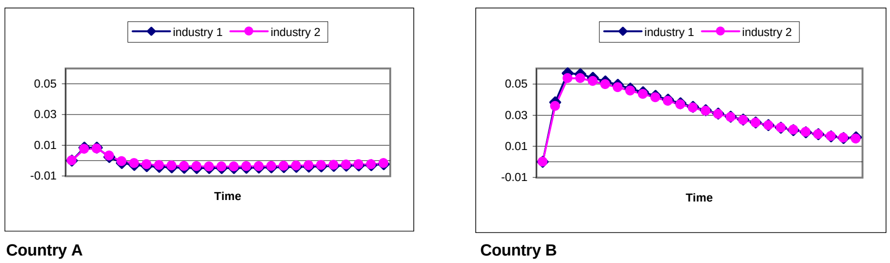


Figure 3.3: Rents

to the higher average world price levels, which is assumed to be quite strong in the less advanced country. But then the prices start falling due to the rapid technical progress. As technical progress is faster in sector 1 (the skill-intensive sector with the higher scope for catching-up), the relative price of good 1 is declining. In the longer run the relative price of good 1 is increasing as the price levels are equalising across countries (by assumption of the price adjustment processes). This goes in hand with the increasing relative wages of skilled workers (which has relatively more impact on the skill-intensive sector 1) and the vanishing impact of technical progress when approaching the technological frontier.

Before studying the labour market effects, we show the evolution of the transitory rents in figure 3.3. Rents are larger in country B as there the technical progress is faster (because

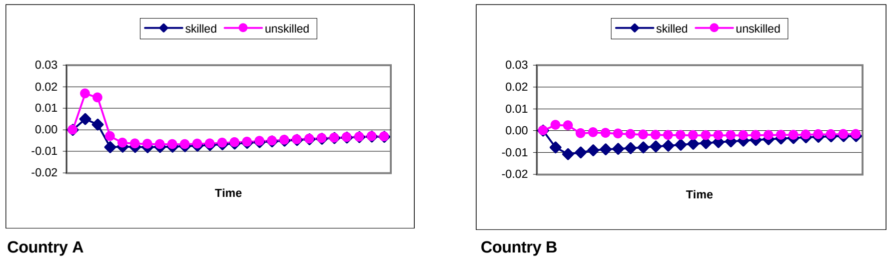


Figure 3.4: Unemployment

of the advantage of technology catching-up) and the impact of the average world price tends to raise price levels in country B. In this model this is a second source of capturing rents, which means a redistribution of income from workers to investors. The difference of rents across sectors is very small. One would expect higher rents in sector 1, as there is more technical progress than in sector 2. But there are two other forces which raises rents in sector 2. First, the adjustment to world prices is higher in sector 2 due to the price structure at the beginning $\frac{p_1^A}{p_1^B} < \frac{p_2^A}{p_2^B}$ and thus raises rents in sector 2. Second, the relative wages of skilled workers are increasing which lowers rents in sector 1. Further rents are small and vanishing very soon in country A, as there is only small technical progress. Rents are becoming even slightly negative, first, because of the impact of world prices and, second, because scarcity in labour supply.

The nominal wage rates are increasing in both countries in the long run although in the initial phase the nominal wage rates decline in both countries for the unskilled workers due to high unemployment rates which arise either because of demand shifts or biased technical progress. The relative wages of skilled workers are increasing in both countries, although much more strongly in country B. The reason for this is, that technical progress is biased against the low-skilled workers, which raises unemployment and thus depresses wages of these groups of workers. In this simulation the wage differentiation across sectors is not particularly strong as the rents are quite similar across sectors.

The evolution of unemployment rates can be seen in figure 3.4. The rise in unemployment rates in country A results from shifts of aggregate demand to country B as we assumed that country B has lower price levels in both goods. In the long run, however, the

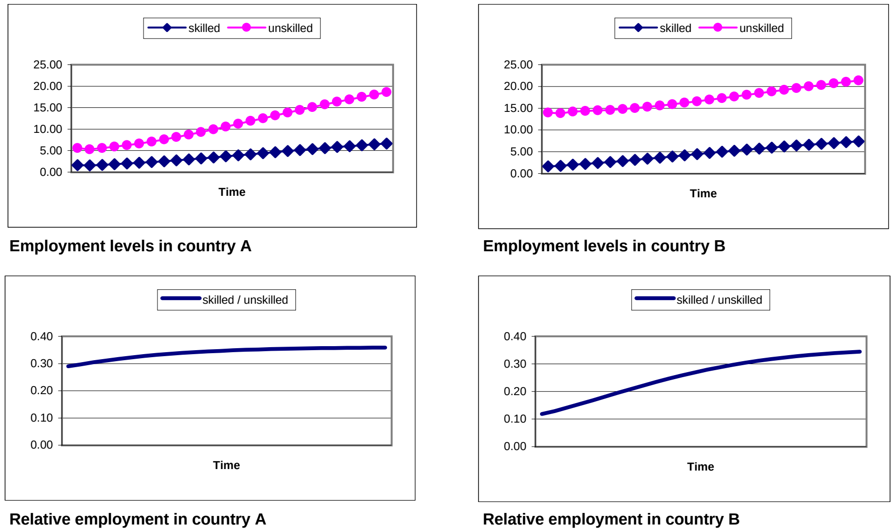


Figure 3.5: Labour demand

economies exhibit overall growth which then even leads to a shortage of labour supply.[15] There are also differences in the structure of unemployment in country A which is quite high for low-skilled workers. These high unemployment rates for low-skilled workers result from the biased technical change and the competition from country B, which is especially strong in the low-skill intensive sector 2 (at least in the initial phase). In country B there is only unemployment for the low-skilled workers due to the biased technical change. As there are high transitory rents, which raises the overall growth rate, the period of unemployment is relatively short in both countries. Here one has to note that the overall growth in country A is due to the spending effects from high rents in country B and the assumption of constant nominal shares, as rents are vanishing in country A quite soon.

Labour demand for both skill types are plotted in figure 3.5. In the long run, labour demand in both countries is rising for both skill types of workers, although there are negative short term effects. The relative employment of skilled to low-skilled workers is

[15] In the numerical simulation we set the adjustment parameter of labour supply to 1. Although this implies quite fast adjustment to labour demand, this adjustment is obviously too slow. As mentioned above, production is not restricted by this shortage but there is an impact on wages via the unemployment term.

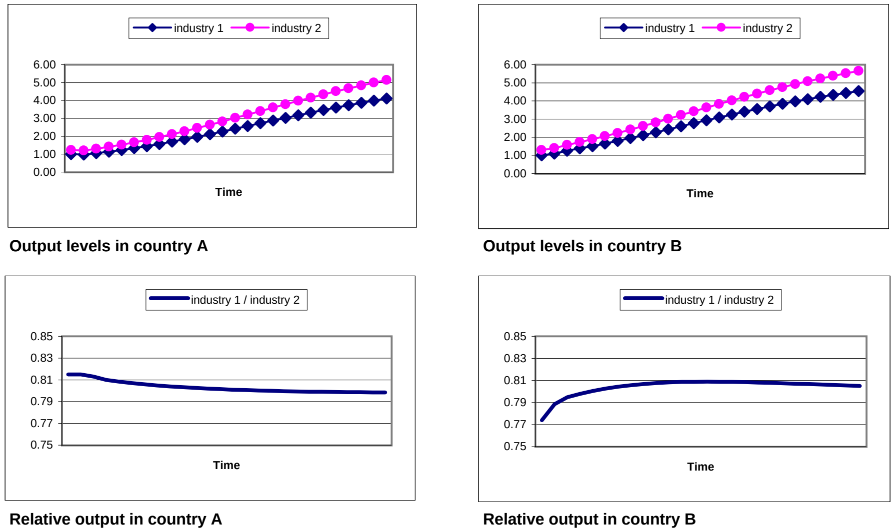


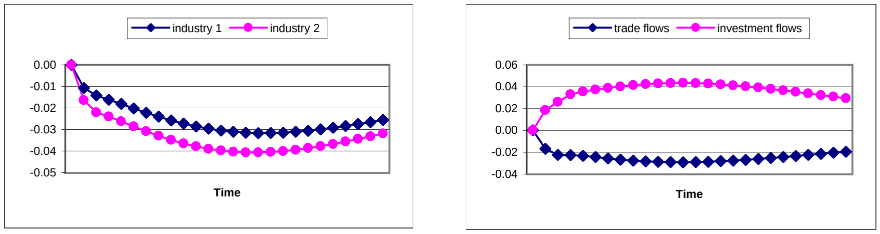
Figure 3.6: Output

rising in both countries. In this model so far, this is only due to a substitution effect on the quantity (demand) side, as we leave out any kind of substitution on the production side (techniques of production).[16] As country B catches up fully to the labour productivity levels in country A, relative demand for skilled workers becomes equal in the long run and the output structure converges between the two countries (see figure 3.6). Output is growing in both economies over the long run due to investment out of the transitory rents. Relative output of good 1 in both countries is getting smaller, as this good is becoming relatively more expensive and thus (consumer) demand shifts to good 2. The relative output of good 1 in country B is even growing in the first phase, as the relative price of this good is getting smaller in this phase.

Finally we discuss the structure of trade and investment flows between the two countries. As the price level is lower in country B the net exports of country A are negative in both sectors.[17] As country A has a comparative advantage in sector 1 (the skill-intensive sector) in the beginning net exports are absolutely higher in sector 2 corresponding to the

[16]In fact a substitution of skilled workers due to the rising relative factor prices w^s/w^u would lower the increase in relative factor demand of skilled workers.

[17]Further the size of the two economies are very similar in terms of quantities and even in terms of gross domestic product.



Net exports of country A (quantity terms) **Balance of trade and investment flows (value terms)**

Figure 3.7: Balance of trade and capital flows

structure of comparative advanteges. Over time however, the structure of comparative advantages changes as one can see in the movements of relative prices in figure 3.2 where even a switchover in comparative advantages takes place. This is reflected in the trade structure as difference between net exports in sector 1 and 2 vanishes and reverses in longer simulation runs.[18]

In values the net exports are equal as the Cobb-Douglas specification implies price elasticities of -1. There are positive capital inflows into country A. This outcome depends on our simple, rather mechanic assumption of the equal nominal investment shares ν_i^{sr} across countries discussed above: As rents are higher in country B than in country A the investments from country B in country A are higher. This is also the reason for the positive overall growth rate in country A. Figure 3.7 presents the resulting trade structure in quantity terms from the viewpoint of country A from which one can see the emerging trade structure and the trade balance and capital balance. The balance of payments is positive as the capital inflows in country A are higher than the (negative) net exports. As mentioned above, the trade and capital flows do not add up to zero as we have not introduced monetary flows.

[18]For this pattern of the dynamics of comparative advantages see Landesmann and Stehrer (2001) and Stehrer and Woerz (2001).

4 Conclusions

This paper presented a dynamic multisectoral model where trade liberalisation and (skill-biased) technical change imply changes in output and employment structures. In this sense, the paper is adressed to a problem which is mainly discussed in the literature using a neo-classical static framework.

Of course there are some drawbacks and thus a potential for improvements in this model which sould be mentioned here. The main problem is the modeling of disequilibrium dynamics. For this reason we introduced some rather ad-hoc behavioural assumptions and adjustment processes at the aggregate level (e.g. for investment behaviour). This issue is one of the next research tasks within this framework. Further we used some simplifying approaches, e.g. for the demand side or labour markets, which could easily be replaced with more sophisticated formulations - a line again to be elaborated in future research. A further issue is the application of simulation studies. As the effects of the exogenous shocks depend on parameter values and especially on combinations of parameter values, the simulations may be complemented by sensitivity analysis, which were not reported in this paper where we have concentrated on the general structure of the model.

On the other hand, the model may be used as a guideline for empirical research to discuss the relative strength of different factors and - which is the advantage of a model like this - the combination of parameter values (which are partly due to institutional settings) on the various variables in an integrated framework. For this reason one may note that the model is formulated in terms which could be compiled empirically (e.g. input-output coefficients, nominal shares in demand, speed of adjustment parameters and elasticities).

Finally we shall summarise potential generalisations and extensions of the model. First, one may extend the behavioural equations as mentioned above. This means e.g. to introduce demand functions which allow for income effects, a better representation of investment behaviour and, finally, substitution effects due to changes in factor prices, e.g. between different skill-type of workers. Second, the model may be generalised in several dimensions, e.g. the number of sectors, the number of skill-types of workers and the number of countries. Here one has to note that - in the way the model is formulated in this paper - there does not exist a dimensionality problem, although the dynamic outcome may not be predicted analytically (for analytical results in the more general model concerning the equilibrium and the steady-state balanced growth path see appendix A). Third, various relationships may be endogenized. For example, the FDI flows may be determined by sector-specific rents. Or the effects of FDI on productivity catching-up could be modeled explicitly. This last point may be generalised to an endogenous determination of the catching-up process itself. Further one could introduce Kaldor-Verdoorn effects which would lead to a path-dependent development process. Fourth, issues of the literature on economic geography may be introduced which lead to interesting developments of spatial structures. This concludes the description of the model and potential extensions and generalisations of the model.

A Equilibrium, balanced and steady-state growth

In this section we discuss the main properties of the model in the steady-state (balanced growth). Here we assume that the technology (input-output matrix) is given and the labour input coefficients are also fixed. Further we assume that wage rates w_i^z are set exogenously and constant. This is the case if the following conditions are satisfied: First, there are no transitory rents, $\mathbf{s}' = \mathbf{0}$ (what will be the case when prices are in equilibrium), or the parameters $\kappa_{s_i} = 0$ for all i, which means that transitory rents does not have an effect on wage rates. Second, wages are equalised across sectors and, third, unemployment has no effect on wages, either because $\kappa_{u^z} = 0$ or labour supply is perfectly elastic.

A.1 Prices, profits, and rents

The equilibrium price vector can be solved for

$$\mathbf{p}' = (1+\pi)\mathbf{b}'\left[\mathbf{I} - (1+\pi)\mathbf{A}\right]^{-1}$$

where $\mathbf{p}'$ is the price vector and $\mathbf{b}'$ denotes the vector of unit labour costs. $\mathbf{I}$ denotes the identity matrix.

The per unit profits in each sector are defined as a mark-up on costs, thus $\mathbf{r}' = \pi\mathbf{c}'$ where $\mathbf{c}'$ denotes per unit costs. Further in disequilibrium there are rents which are defined as: $\mathbf{s}' = \mathbf{p}' - (1+\pi)\mathbf{c}'$. In equilibrium these rents are zero as one can show by inserting the equilibrium price vector. In the following we define a vector $\mathbf{v}'$ which adds up profits and rents $\mathbf{v}' = (\mathbf{r}' + \mathbf{s}') = \mathbf{p}' - \mathbf{c}'$. In the case that these transitory rents are not zero we have to assume that $\kappa_{s_i} = 0$ for all i if $s_i > 0$ for guaranteeing constancy of wages as mentioned above. By this assumption we can also show the more general case where profits and rents need not to be equalised across sectors, either because of different s_i or long-term sector-specific mark-ups π_i.

A.2 The quantity system

Next we discuss the quantity system. Here we have to assume that $\dot{\mathbf{p}} = \mathbf{0}$. Thus, the results presented below assume only stable prices, although these need not be equilibrium prices. Demand consists of three different components: First there is demand for intermediate goods used in production, $\mathbf{Aq}$, where $\mathbf{q}$ denotes the vector of quantities. Second there is a matrix of demand out of profits and rents (used for investment).

γ_{ij} denotes the share of rent income generated in sector i and invested in sector j with $\sum_j \gamma_{ij} = 1$ for each sector i. β_{jk} is the share of expenditure for buying investment good k when investing in sector j. Again the condition $\sum_k \beta_{jk} = 1$ has to be satisfied for all sectors j. Demand for investment goods can then be written as

$$\mathbf{D}_v\mathbf{q} = \begin{pmatrix} \frac{v_1}{p_1}\sum_j \gamma_{1j}\beta_{j1} & \cdots & \frac{v_N}{p_1}\sum_j \gamma_{Nj}\beta_{j1} \\ \vdots & \ddots & \vdots \\ \frac{v_1}{p_N}\sum_j \gamma_{1j}\beta_{jN} & \cdots & \frac{v_N}{p_N}\sum_j \gamma_{Nj}\beta_{jN} \end{pmatrix}\mathbf{q} = \mathbf{q}^I$$

A typical element of the matrix $\frac{v_i}{p_k}\sum_j \gamma_{ij}\beta_{jk}$ thus denotes the demand for investment goods in sector k from sector i investing in sector j. The nominal share β_{jk} is determined by the technological coefficients a_{kj} and the prices p_k. As we have a model with fixed coefficients of production we get $\beta^*_{jk} = \frac{p_k a_{kj}}{\mathbf{p}'\mathbf{a}_{*j}}$. Further we assume that the investment decisions γ_{ij} are equal across industries. Using these assumptions gives

$$\mathbf{D}_v\mathbf{q} = \begin{pmatrix} \frac{v_1}{p_1}\sum_j \gamma_j \frac{p_1 a_{1j}}{\mathbf{p}'\mathbf{a}_{*j}} & \cdots & \frac{v_N}{p_1}\sum_j \gamma_j \frac{p_1 a_{1j}}{\mathbf{p}'\mathbf{a}_{*j}} \\ \vdots & \ddots & \vdots \\ \frac{v_1}{p_N}\sum_j \gamma_j \frac{p_N a_{Nj}}{\mathbf{p}'\mathbf{a}_{*j}} & \cdots & \frac{v_N}{p_N}\sum_j \gamma_j \frac{p_N a_{Nj}}{\mathbf{p}'\mathbf{a}_{*j}} \end{pmatrix}\mathbf{q} = \mathbf{q}^I$$

We shall show below that the economy exhibits a balanced growth path if $\gamma_j^* = \frac{\mathbf{p}'\mathbf{a}_{*j} q_j}{\mathbf{p}'\mathbf{A}\mathbf{q}}$. Inserting gives

$$\mathbf{D}_v \mathbf{q} = \frac{1}{\mathbf{p}'\mathbf{A}\mathbf{q}} \begin{pmatrix} v_1 \sum_j a_{1j} q_j & \dots & v_N \sum_j a_{1j} q_j \\ \vdots & \ddots & \vdots \\ v_1 \sum_j a_{Nj} q_j & \dots & v_N \sum_j a_{Nj} q_j \end{pmatrix} \mathbf{q} = \frac{\mathbf{v}\mathbf{q}}{\mathbf{p}'\mathbf{A}\mathbf{q}} \mathbf{A}\mathbf{q} = \mathbf{q}^I$$

The third source of demand stems from wage income. Consumption expenditures out of wages are denoted in matrix form

$$\mathbf{D}_w \mathbf{q} = \begin{pmatrix} \alpha_1 \frac{\sum_z w_1^z a_{l1}^z}{p_1} & \dots & \alpha_1 \frac{\sum_z w_N^z a_{lN}^z}{p_1} \\ \vdots & \ddots & \vdots \\ \alpha_N \frac{\sum_z w_1^z a_{l1}^z}{p_N} & \dots & \alpha_N \frac{\sum_z w_N^z a_{lN}^z}{p_N} \end{pmatrix} \mathbf{q} = \mathbf{q}^F$$

α_i are the nominal shares in consumption with $\sum_i \alpha_i = 1$. The specific assumption in this formulation is that workers are maximising a Cobb-Douglas utility function, which is linear-homogenous and homothetic. This means that all workers have the same (constant) nominal shares of consumption. A more more general specification of the demand (e.g. dependent on real income levels and prices) could be used here. For given wage rates and prices the nominal shares $\alpha_i^z(w_i^z, \mathbf{p})$ would then also be constant although differing across skill types of workers and, in the case of wage differentiation across sectors, differ across skill types and sectors. A typical element in the matrix would then be $\sum_z \alpha_{i,j}^z \frac{w_i^z a_{li}^z}{p_j}$ where $\sum_j \alpha_{i,j}^z = 1$. We do not explore this general case here. Total demand is the sum of these three components

$$\mathbf{q}^D = \mathbf{A}\mathbf{q} + \mathbf{D}_v \mathbf{q} + \mathbf{D}_w \mathbf{q} = (\mathbf{A} + \mathbf{D}_v + \mathbf{D}_w) \mathbf{q}$$

In equilibrium we must have $\mathbf{q}^D = \mathbf{q}$ and thus the expression above

$$(\mathbf{A} + \mathbf{D}_v + \mathbf{D}_w - \mathbf{I}) \mathbf{q} = \mathbf{0}$$

This is a linear-homogenous system which has a non-trivial solution, $\mathbf{q} \neq \mathbf{0}$, if the determinant of $(\mathbf{A} - \mathbf{I} + \mathbf{D}_v + \mathbf{D}_w)$ equals zero which means that the columns or rows are linearly dependent.[19]

Premultiplying $(\mathbf{A} - \mathbf{I} + \mathbf{D}_v + \mathbf{D}_w)$ with $\mathbf{p}'$ yields the condition $\mathbf{p}'(\mathbf{A} + \mathbf{D}_v + \mathbf{D}_w) = \mathbf{p}'\mathbf{I}$. This can be rewritten as

$$\mathbf{p}'(\mathbf{A} - \mathbf{I} + \mathbf{D}_v + \mathbf{D}_w) = \mathbf{p}'\mathbf{D}_v - \mathbf{p}'[\mathbf{I} - (\mathbf{A} + \mathbf{D}_w)] = \mathbf{0}'$$

Using $\mathbf{p}'\mathbf{D}_v = \mathbf{v}'$ and $\mathbf{p}'\mathbf{D}_w = \mathbf{b}'$ and inserting gives

$$\mathbf{v}' - [\mathbf{p}' - (\mathbf{p}'\mathbf{A} + \mathbf{b}')] = \mathbf{v}' - \mathbf{v}' = \mathbf{0}'$$

which shows the existence of a non-trivial solution. Accordingly to the Perron-Frobenius theorems the maximum eigenvalue of $(\mathbf{A} + \mathbf{D}_v + \mathbf{D}_w)$ equals 1 of which the components of the associated eigenvector are non-negative.

The dynamics of the supply of goods is modeled as a system of differential equations

$$\dot{\mathbf{q}} = (1+g)[\mathbf{I} - \mathbf{A}]^{-1} (\mathbf{D}_v + \mathbf{D}_w) \mathbf{q} - \mathbf{q}$$

Inserting for $(\mathbf{D}_v + \mathbf{D}_w) = (\mathbf{I} - \mathbf{A})$, which is satisfied in equilibrium, gives

$$\dot{\mathbf{q}} = (1+g)[\mathbf{I} - \mathbf{A}]^{-1} [\mathbf{I} - \mathbf{A}] \mathbf{q} - \mathbf{q} = g\mathbf{q}$$

Thus the quantity system grows at a constant rate g (steady-state balanced growth path).[20]

[19] Please note, that this condition is analogue to the condition of the existence of a solution in the closed Leontief model.

[20] In equilibrium this formulation is equivalent with

$$\dot{\mathbf{q}} = [\mathbf{I} - (1+g)\mathbf{A}]^{-1} (1+g)\mathbf{D}_w \mathbf{q} - \mathbf{q}$$

as used e.g. in Pasinetti (1977).

We have to analyse the relationship between the demand (and supply) for investment goods $\mathbf{D}_v\mathbf{q}$ and the vector of growth rates $\mathbf{g}$. The system is constant (in the case $v_i = 0$, or $\pi_i = 0$ and $s_i = 0$ for all $i = 1, \ldots, N$) or is growing with

$$\mathbf{g} = \mathbf{Q}^{-1}\mathbf{A}^{-1}\mathbf{q}^I = \frac{\mathbf{vq}}{\mathbf{p}'\mathbf{Aq}}\mathbf{Q}^{-1}\mathbf{A}^{-1}\mathbf{Aq} = \frac{\mathbf{vq}}{\mathbf{p}'\mathbf{Aq}}\boldsymbol{\iota}$$

where $\mathbf{Q}$ denotes a diagonal matrix with q_i at the diagonal and $\boldsymbol{\iota}$ is a vector with ones. In equilibrium (i.e. with $\mathbf{s} = \mathbf{0}$ or at prices $\mathbf{p}^*$) this can be reformulated as

$$g^* = \frac{\mathbf{r}'\mathbf{q}}{\mathbf{p}'\mathbf{Aq}} = \frac{\mathbf{r}'\mathbf{q}}{\mathbf{p}'\mathbf{q}}\frac{\mathbf{p}'\mathbf{q}}{\mathbf{p}'\mathbf{Aq}} = \frac{\pi\mathbf{c}'\mathbf{q}}{(1+\pi)\mathbf{c}'\mathbf{q}}\frac{\mathbf{p}'\mathbf{q}}{\mathbf{p}'\mathbf{Aq}} = \frac{\pi}{1+\pi}\frac{\mathbf{p}'\mathbf{q}}{\mathbf{p}'\mathbf{Aq}}$$

If this condition is satisfied then the economy is growing in equilibrium exactly at the rate g^*. In the case that $\pi^{max} = \frac{1}{\lambda_{\mathbf{A}}^{max}} - 1$, i.e. the maximum profit rate, and under the assumption that the whole surplus is reinvested this gives

$$g^{max} = \frac{\pi^{max}}{1+\pi^{max}}\frac{1}{\lambda_{\mathbf{A}}^{max}} = \pi^{max}$$

This is a simple form of the von Neumann result in which the economy grows at the maximum rate of profit.

A.3 Labour demand and supply

Labour demand is then modeled simply by $L^{D_z} = \mathbf{a}_l^{z'}\mathbf{q}$ for each skill group z. The formulation implies that labour supply of each skill-type also has to grow at a rate g or g^*, respectively.

A.4 Derivation of demand for investment goods and growth rates for integrated economies

Finally we discuss shortly the case of integrated economies but restrict ourself to the discussion of long-run relationships.

A.4.1 Investment demand in country c

In the text the following equation (here partly written in matrix notation) was stated for demand of goods in country c for investment purposes in countries s financed by countries r.

$$(\mathbf{q}^I)^c = \sum_s\sum_r \frac{\sum_k \nu_k^{rs}\xi_k^{rc}v_k^r q_k^r}{(\mathbf{p}')^s\mathbf{A}^s\mathbf{q}^s}\mathbf{A}^s\mathbf{q}^s$$

Given the derivation of demand for investment goods in the closed economy we can easily interpret this equation. First, however, we have to note that this formulation assumes that $\beta_{jk}^{*sc} = \frac{p_k^c a_{kj}^s}{(\mathbf{p}')^c\mathbf{a}_{*j}^s}$ and $\gamma_j^{*s} = \frac{(\mathbf{p}')^c\mathbf{a}_j^s q_j^s}{(\mathbf{p}')^s\mathbf{A}^s\mathbf{q}^s}$ which takes into account that prices in country s where goods are purchased and country c where the goods are invested can be different. Then the formula above can be interpreted in line with the closed economy case as $\sum_{r,k}\nu_k^{rs}\xi_k^{rc}v_k^r q_k^r$ is the nominal sum of profits and rents of countries r investing in a specific country s and purchasing these goods in country c. As in the closed economy case the sum of profits and rents is distributed in a way such that the growth rate in country s is maximised but here one takes into account that prices in country c where goods are purchased are different.

A.4.2 Physical investment and determination of the growth rate in country c

Departing from these calculations of demand one can calculate the physical investment in a specific country c.

$$\left(\mathbf{q}^I\right)^c = \left((\mathbf{p}')^c \mathbf{A}^c \mathbf{q}^c\right)^{-1} \left[\sum_r \sum_s \sum_k \nu_k^{rc} \xi_k^{rs} v_k^r q_k^r\right] \mathbf{A}^c \mathbf{q}^c = \left((\mathbf{p}')^c \mathbf{A}^c \mathbf{q}^c\right)^{-1} \left[\sum_{k=1}^{N} \sum_{r=1}^{C} \nu_k^{rc} v_k^r q_k^r\right] \mathbf{A}^c \mathbf{q}^c$$

where we use $\sum_s \xi_k^{rs} = 1$. Using $\mathbf{g}^c = (\mathbf{Q}^c)^{-1}(\mathbf{A}^c)^{-1}(\mathbf{q}^I)^c$ and inserting for $(\mathbf{q}^I)^c$ gives

$$\mathbf{g}^c = \left((\mathbf{p}')^c \mathbf{A}^c \mathbf{q}^c\right)^{-1} \left[\sum_{k=1}^{N} \sum_{r=1}^{C} \nu_k^{rc} v_k^r q_k^r\right] \boldsymbol{\iota}$$

which - as in the closed economy case - shows that the growth rate is balanced in equilibrium and is determined by the ratio of nominal profits and rents (in this case of all countries investing in country c) over the nominal value of the variable capital.

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